

Performance and Optimization Metrics for Interferometric Radar

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ABSTRACT

As the demands for interferometric radar products increase, so does the need for developing and building new instruments to provide digital elevation maps with higher accuracy and lower costs. Additionally, the variety of instruments is increasing, and for this reason, it is important to develop a methodology for analyzing and comparing systems using simple analytic formulas.

This paper provides a set of analytic equations that can be used to estimate system performance or to assist in the design of a system in that individual sources of height error can be balanced to minimize cost and maximize efficient use of resources.

INTRODUCTION

The impetus for this work began with the development of GeoSAR, an airborne interferometric system now being constructed at JPL. Performance requirements for the two frequency system are to achieve 2 m relative (and 1 m absolute) height accuracy with the X-band system and 4 m relative height accuracy with the 160 MHz bandwidth P-band system. To achieve this goal, the allowed height errors are budgeted to different principal sources to provide direct hardware performance specifications. After the initial design is completed, the performance metrics can be used again to relax/enforce specific performance constraints.

RMS HEIGHT ACCURACY EQUATIONS

The problem with the current design equations is that they are for a single point in the swath. It is our purpose to build the system to meet an average height error requirement rather than a point by point height error requirement. Thus, errors in the far range may exceed specification (within reason) as long as those errors in the near range are small enough to compensate. It was noticed that most of the error components could be integrated over look angle and therefor a swath-averaged error can be analytically calculated.

The error for any point along the swath may be estimated by summing together the errors induced by separate components of the system:

$$\sigma_z^2 = A_\phi^2 \sigma_\phi^2 + A_B^2 \sigma_B^2 + A_\xi^2 \sigma_\xi^2 + A_H^2 \sigma_H^2 + A_\lambda^2 \sigma_\lambda^2 + A_\rho^2 \sigma_\rho^2$$

where ϕ refers to the interferometric phase, B is the baseline, ξ is the baseline tilt angle, H is the platform altitude, λ is the wavelength and ρ is the distance to the

target. Integrating the square root of this quantity over the swath will give the average error. This integral is difficult to determine analytically, while an integral of the individual error components is a fairly straightforward task and will provide an upper limit to average error. An upper and lower limit to the height error across the swath can be determined by

$$\left\{ \left[\int A_\phi \sigma_\phi dy \right]^2 + \left[\int A_B \sigma_B dy \right]^2 + \left[\int A_\xi \sigma_\xi dy \right]^2 + \left[\int A_H \sigma_H dy \right]^2 + \left[\int A_\lambda \sigma_\lambda dy \right]^2 + \left[\int A_\rho \sigma_\rho dy \right]^2 \right\}^{1/2} \leq \int \left[\sigma_z^2 \right]^{1/2} dy \leq \int A_\phi \sigma_\phi + A_B \sigma_B + A_\xi \sigma_\xi + A_H \sigma_H + A_\lambda \sigma_\lambda + A_\rho \sigma_\rho dy$$

The objective of this work is to determine the integrals of the components of $A_\phi \sigma_\phi$ and to find an upper bound for the average height error across the swath.

PHASE NOISE, Φ

The interferometer measures the product of the two electric fields received by two antennas separated by a baseline.

$$I = \langle E_1 E_2^* \rangle \approx \frac{1}{L} \sum_{l=1}^L E_1 E_2^*$$

Phase noise results from a vector noise component appearing in the data stream (of each antenna) in addition to the desired signal. This may be modeled by

$$E_1 = E_{10} + E_{11} + E_{12} + \dots$$

$$E_2 = E_{20} + E_{21} + E_{22} + \dots$$

where the second component of the subscript may be considered to be signal (hereby a zero subscript) or one of any number of possible error sources (ambiguities, thermal noise, multipath, etc., denoted by a non-zero subscript). Some of the components of the electric field are decorrelated between different looks, and thus their effect reduces with the number of looks used in the average (i.e. $\langle E_{1i} E_{2j} \rangle = 0$).

Other components of the electric field do not decorrelate and these sources do not reduce with the number of looks. As a general treatment, the errors generated by phase noise can be most easily addressed by categorizing them into three components: thermal noise, correlated phase noise and uncorrelated phase noise:

$$\sigma_{\phi, tot}^2 = \sigma_{\phi, therm}^2 + \sigma_{\phi, uncorr}^2 + \sigma_{\phi, corr}^2$$

where the subscripts refer to thermal noise (i.e. from the receiver), and noise related to the scene being imaged. In addition, each of the subcomponents (i.e. therm, uncorr, or corr) may be a combination of sources.

A simple model relating phase noise to signal to noise ratio of a single look (SNR_1) is given by [Ernesto and Martin, 1992] as

$$\sigma_{\phi, N}^2 = \frac{1}{N * SNR_l}$$

which is appropriate for a large signal to noise ratio and the number of looks being four or greater. In this analysis, for the sake of simplicity, this formula will also be used to estimate the effect of single look, or correlated, phase noise. Multiple sources of phase noise may be combined by adding their contribution to the total height error variance

$$\sigma_{\phi, tot}^2 = \frac{1}{N} \left(\frac{1}{SNR_{l, therm}} + \frac{1}{SNR_{l, uncorr}} + \dots \right) + \left(\frac{1}{SNR_{l, corr}} + \dots \right)$$

In general, it is necessary to differentiate between correlated and uncorrelated phase noise sources. To give a foundation for this treatment we form the complex product

$$\begin{aligned} \langle E_1 E_2^* \rangle &= \langle E_{10} (E_{20} + E_{21} + E_{22} + \dots)^* + \\ &\quad E_{11} (E_{20} + E_{21} + E_{22} + \dots)^* + \\ &\quad E_{12} (E_{20} + E_{21} + E_{22} + \dots)^* + \dots \rangle \end{aligned}$$

where cross terms such as $\langle E_{1i} E_{2j}^* \rangle \approx 0$ when $i \neq j$. There are two questions that must be answered for each component in the above equation:

- i.) is E_{1l} uncorrelated to E_{2l} ? (i.e. $\langle E_{1l} E_{2l}^* \rangle = 0$)
- ii.) is E_{1l} uncorrelated to $E_{2(l+1)}$? (i.e. $\langle E_{1l} E_{2(l+1)}^* \rangle = 0$)

when l represents the look number. As will be discussed further, the first type of decorrelation relates to decorrelation between the antenna pair (such as baseline decorrelation, wrong-side ambiguities, etc.). The second type of refers to time varying signals that decorrelate (such as thermal receiver noise or possible quantization noise). The only difference in the way that these two noise sources are treated is if the number of looks may be used to increase the signal to noise ratio. For GeoSAR, the number of looks is minimally 50 making the SNR improvement a substantial 17 dB.

Uncorrelated Phase Noise

Thermal noise is one example of uncorrelated phase noise. It has received special treatment here because the signal to noise ratio changes appreciably as a function of position along the swath. Other phase noise sources, such as multipath, wrong-side ambiguities, and quantization noise, can be approximated as being constant across the swath. For these noise sources, the average error across the swath may be analytically determined, as in

$$\begin{aligned} \overline{err_{\phi, uncorr}} &= \frac{1}{SW} \int \sigma_{\phi, uncorr} \frac{\lambda H \sin \theta}{a 2 \pi B \cos^2 \theta} \frac{H}{\cos^2 \theta} d\theta \\ &= \frac{1}{SW} \frac{\lambda H^2}{a 2 \pi B} \left[\frac{1}{\cos^4 \theta_{max}} - \frac{1}{\cos^4 \theta_{min}} \right] \frac{1}{\sqrt{N} \sqrt{SNR_{l, uncorr}}} \end{aligned}$$

Thermal Noise

A certain amount of noise is inherent to the receiver chain and is proportional to the physical temperature of the system itself. This particular form of noise is range and

look angle dependent in that the strength of the return signal varies as a function of these parameters. The thermal signal to noise ratio may be determined by the radar equation (Ulaby, 1986; Curlander, ??) for a distributed target using a synthetic aperture radar. The most convenient form of this equation for this application is

$$SNR_{therm}^{-1} = \frac{P_{noise}}{P_{recv}}, \text{ where } P_{noise} = kTB / N_{oversamp},$$

$$\text{and } P_{recv} = P_{trans} \frac{G^2(\theta) \lambda^2 \sigma^o}{(4\pi)^3 \rho^4} \frac{c \tau_p}{2 \sin \eta} \frac{\lambda \rho}{L_{az}} F_{loss}$$

$N_{oversamp}$ is the degree of azimuth oversampling (related to the PRF and the aircraft velocity) and $G(\theta) = G_{bore} F(\theta)$ is the elevation plane antenna pattern. For GeoSAR, two frequency bands will be used, P-band (360 Mhz center freq.) and X-band (10 Ghz). At P-band a single element circular ring (diameter 17") will be used in the elevation plane (an array will be employed for the azimuth plane) and for the X-band antenna subsystem will be a slotted waveguide. The elevation-plane gain function (power) for both of these systems may be approximated as

$$F(\theta) = \begin{cases} \cos^4(\theta_{bore} - \theta) & \text{P-band} \\ \text{sinc}^2(\pi \frac{L_{el}}{\lambda} \sin(\theta_{bore} - \theta)) & \text{X-band} \end{cases}$$

where L_{el} is the height of the X-band aperture in the elevation plane. The average error across the swath due to phase noise is

$$\begin{aligned} \overline{err_{\phi}} &= \frac{1}{SW} \int A_{\phi} \sigma_{\phi} dy \quad A_{\phi} = \frac{\lambda H \sin \theta}{a 2 \pi B \cos^2 \theta} \quad dy = \frac{H}{\cos^2 \theta} d\theta \\ \overline{err_{\phi, therm}} &= \left[\frac{32 \pi H^7 L_{az}}{N a^2 B^2 \lambda c \tau_p G_{bore}^2 F_{loss} P_{trans}} \frac{P_{noise}}{F_{loss} P_{trans}} \right]^{1/2} \frac{1}{SW} \int \left[\frac{\sin^2 \theta}{\sigma^o F^2(\theta) \cos^{11} \theta} \right]^{1/2} d\theta \end{aligned}$$

where the latter integral may be computed numerically with the option that target backscatter may or may not be a function of the angle of incidence. The look angle limits of the integral can be determined beforehand, dependent on the platform altitude, to assure that the swath meets the correct width specifications. Given a minimum look angle, θ_{min} , and a swath width of SW, the maximum look angle is

$$\theta_{max} = \text{atan}[SW / H + \tan \theta_{min}].$$

For a minimum look angle of 27 degrees, the upper limit, is

$$\theta_{max} = \begin{cases} 56.5^\circ & H = 10 \text{ km} \\ 69^\circ & H = 5 \text{ km} \end{cases}$$

Correlated Phase Noise

Similarly, some phase noise sources may be correlated between successive radar pulses. These noise sources, while not fluctuating as a function of position in the swath, are scene dependent in that the source of phase noise comes from the same location in the scene between successive looks. Using the same formalism as the uncorrelated phase noise, the average error across the swath due to correlated phase noise is

$$\overline{err_{\phi,corr}} = \frac{1}{SW} \frac{\lambda H^2}{a 2 \pi B} \left[\frac{1}{\cos^4 \theta_{\max}} - \frac{1}{\cos^4 \theta_{\min}} \right] \frac{1}{\sqrt{SNR_{i,corr}}}$$

The only difference between this error source and the uncorrelated phase noise error source is that this one may not be reduced by increasing the number of looks. For this reason, this error source is particularly problematic.

BASLINE ERROR, B

Baseline error is induced by fluctuations in the baseline length that cannot be measured (i.e. below the accuracy of the baseline metrology subsystem). The average error across the swath is computed by

$$\overline{err_B} = \frac{1}{SW} \int A_B \sigma_B dy \quad A_B = \frac{-H \tan^2 \theta}{B} \quad dy = H \sec^2 \theta d\theta$$

$$\overline{err_B} = \frac{1}{SW} \frac{-H^2 \sigma_B}{3B} [\tan^3 \theta_{\max} - \tan^3 \theta_{\min}]$$

BASLINE TILT ANGLE ERROR, ξ

Baseline tilt is the angle with respect to the horizontal that the baseline is oriented. Errors in the determination of the baseline tilt is a function of a combination of the embedded GPS/INS (EGI) and the baseline metrology subsystems. The average error across the swath for this particular error source is

$$\overline{err_{\xi}} = \frac{1}{SW} \int A_{\xi} \sigma_{\xi} dy \quad A_{\xi} = H \tan \theta \quad dy = H \sec^2 \theta d\theta$$

$$\overline{err_{\xi}} = \frac{1}{SW} \int_{\theta_{\min}}^{\theta_{\max}} H^2 \tan \theta \sec^2 \theta \sigma_{\xi} d\theta = \frac{H^2 \sigma_{\xi}}{2SW} [\tan^2 \theta_{\max} - \tan^2 \theta_{\min}]$$

ALTITUDE DETERMINATION ERROR, H

Platform altitude is determined by the GPS/INS subsystem. Errors in measuring the platform altitude feed one to one into the height error of the system. The average error across the swath is simply the average error of the GPS/INS system in determining platform altitude,

$$\overline{err_H} = \frac{1}{SW} \int A_H \sigma_H dy \quad A_H = 1 \quad \overline{err_H} = \sigma_H$$

RANGE ERROR, ρ

Errors/inaccuracies in the system timing is a source of uncertainty for determining the range to the target. These uncertainty translates into a range error which effects the average error across the swath by

$$\overline{err_{\rho}} = \frac{1}{SW} \int A_{\rho} \sigma_{\rho} dy \quad A_{\rho} = \cos \theta \quad dy = H \sec^2 \theta d\theta$$

$$\overline{err_{\rho}} = \frac{H}{SW} \sigma_{\rho} \ln \left| \frac{\tan(\theta_{\max} / 2 + \pi / 4)}{\tan(\theta_{\min} / 2 + \pi / 4)} \right|$$

WAVELENGTH ERROR, λ

The stable local oscillator for the system is typically accurate to one part in ten million (i.e. 10^{-7}). The slight variation in frequency translates into a wavelength error

$$\sigma_{\lambda} = \frac{c}{f^2} \sigma_f$$

which can then be used for determining the average height error across the swath induced by wavelength error.

$$\overline{err_{\lambda}} = \frac{1}{SW} \int A_{\lambda} \sigma_{\lambda} dy \quad A_{\lambda} = \frac{H \tan^2 \theta}{\lambda} \quad dy = H \sec^2 \theta d\theta$$

$$\overline{err_{\lambda}} = \frac{H^2 \sigma_{\lambda}}{3 \lambda SW} [\tan^3 \theta_{\max} - \tan^3 \theta_{\min}]$$

At this stage the analytic expressions have been developed for determining the average height error created by errors for each of the fundamental accuracies in the measurement of system phase, baseline length and tilt, altitude, wavelength, and range. The goal now will be to use this set of equations for determining the upper bound to the height error across the swath as a function of the variables that may be altered during the system design (i.e. transmitted power, baseline length, etc). Those components of the equations where the variables that will remain fixed (wavelength, altitude, maximum and minimum look angle), may be evaluated beforehand. Thus a set of expressions can be derived that explicitly relate the height error to specific components of the system design.

Design Example

The following represents the application of the developed set of equations in the design of the P-band GeoSAR system. All of the expressions may be evaluated using a simple calculator with the exception of the thermal noise component. In this case, the integral of the antenna gain function and other angle dependent factors must be numerically calculated. This calculation however will not change appreciably as a function of the system parameters (i.e. the only system design variable is the antenna pattern in the azimuth direction and the antenna look angle), and thus, it may be assumed to remain static during the initial design and evaluation phase. Using $F(\theta) = \cos^4(\theta_{\text{bore}} - \theta)$, the value of the integral as a function of platform altitude (for a swath width of 10 km) is shown in Figure 1.

There are three separate methods for calculating the swath averaged height error (equations given on the front page of this document). The first method, giving the lower bound to the error, is accomplished by finding the root mean square of the average error due to each of the error source (i.e. take the square root of the sum of the squares of $\overline{err_{\phi}}$, $\overline{err_H}$, ...etc.). The "exact" formulation comes from numerically integrating the total height error evaluated each point in the swath, and the upper limit is given by the simple sum of the average height errors across the swath. The result of these calculations is summarized in the following table and in Figure 2.

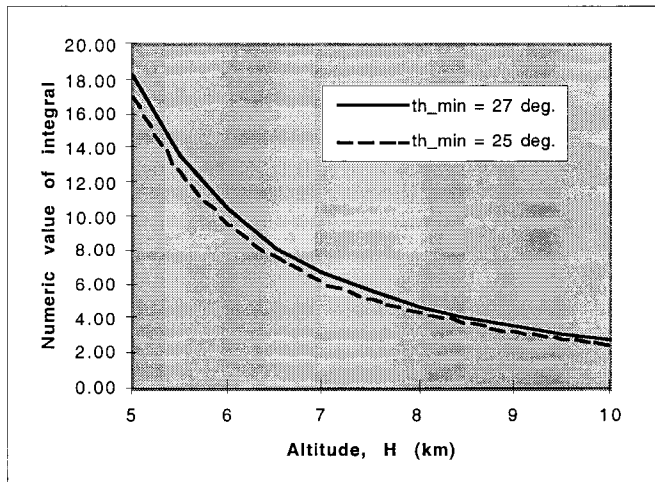


Figure 1: Numerical integration results for calculating the look angle dependent terms for the phase noise component of average error across the swath. $F(\theta) = \cos^4(\theta_{\text{bore}} - \theta)$ is used for the antenna gain function with an boresight of sixty degrees. The minimum look angle is a design specification given by th_{min} .

Error Source	Measurement Error	Avg Height Error at 5km (10km)
Phase Noise	(varies in swath)	0.14 (0.24)
Baseline Length	0.7 mm	0.46 (0.39)
Baseline Tilt Angle	15 arc-seconds	0.55 (0.73)
Slant Range	0.6 m	0.35 (0.43)
Altitude	0.1 m	0.1 (0.1)
Wavelength	1×10^{-7} Hz	0.0 (0.0)
Minimum bound		0.8 (1.0)
"Exact calculation"		1.3 (1.6)
Maximum bound		1.6 (1.9)

Table 1 Expected average height errors for the GeoSAR P-band interferometer based on assumptions of how accurately certain system parameters can be measured. The minimum bound is based on the rms average of the swath averaged errors, the "exact" calculation reflects the numerical integration of the rms average for each point in the swath and the maximum bound is the sum of the swath averaged height errors (see Figure 3 below).

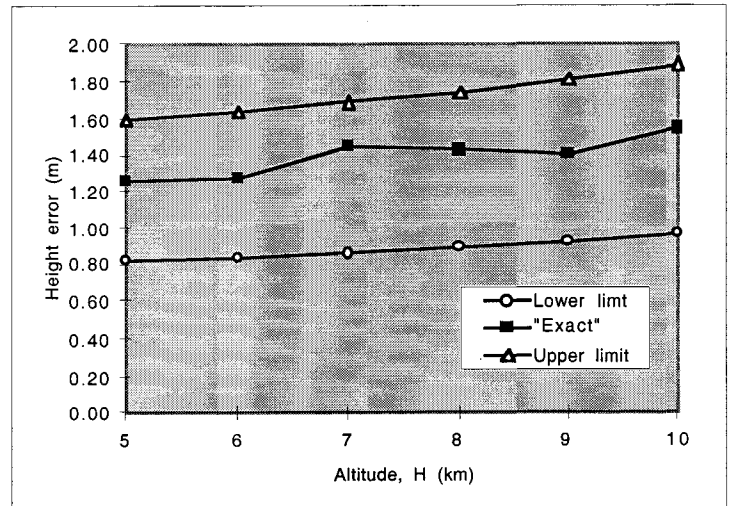


Figure 2: Average height error calculations. Shown are the three methods of calculating average height error across the swath: i) rms average of individual swath averaged errors (Lower limit), ii) swath average of rms errors ("Exact") and iii) sum of swath averaged errors (Upper limit).

Acknowledgements. The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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